Improved Autoregressive (AR) Signal Modeling for FDTD Resonance Estimation

A. Lauer and I. Wolff
Institute of Mobile and Satellite Communication Techniques,
D–47475 Kamp-Lintfort, Germany, e-mail lauer@imst.de

Abstract—**Standard Autoregressive Signal Processing is very sensitive to the model system's order, instabilities can occur and make the model unusable. To overcome this drawback, in this paper a mixed DFT/AR (DAR) method is presented using frequency domain AR evaluation, which is insensitive to instable poles of the AR model. Numerical examples for the good performance of the algorithm are presented.**

I. Introduction

Based on growing PC performance, the Finite Difference Time Domain (FDTD, [1]) method for the numerical integration of Maxwell's Equations is more and more used by RF developers as a general purpose analysis and optimization tool.

Results in the frequency domain are normally generated from time series using the Discrete Fourier Transform (DFT). If e. g. high Q resonant systems are under consideration, long time series are needed for accurate results, which requires long FDTD computation times.

Thus various Signal Processing [2]–[4] algorithms have been developed for accurate frequency domain signal estimation from short time series. Autoregressive Signal Models (AR) [3], [4], assuming each time sample to be a linear combination of n previous samples, are simple and fast. However, [3], [4] do report a strong sensitivity of AR(n) systems with regard to the system order n, making high order systems difficult to use.

These problems are due to the time series extrapolation technique [4], which is used in conjunction with the DFT algorithm. If the AR–model yields instable poles, the extrapolated time series can not be properly treated with the DFT.

This paper overcomes this problem by direct frequency domain AR evaluation in combination with DFT of the FDTD generated time signal.

Section II explains the AR model used. Section III introduces the z–Domain AR difference equation solution and shows, how AR and DFT work together in the frequency domain signal generation. In Section IV numerical results are presented in comparison to the standard AR method.

II. The Auto–Regressive signal model

If s_k is a properly sampled discrete time domain signal, the AR(n)–model assumes

$$s_k = \sum_{\nu=1}^{n} a_\nu s_{k-\nu}, \qquad (1)$$

where the a_ν are the unknown model parameters.
If m samples from the end of the FDTD calculated signal s_k, $k \in [0, M]$ are used for model parameter estimation,

$$s_{M-\mu+1} = \sum_{\nu=1}^{n} a_\nu s_{M-\mu-\nu+1}, \qquad \mu \in [1, m-n] \qquad (2)$$

is a system

$$\mathbf{A}\vec{a} = \vec{b}, \qquad (3)$$
$$b_\mu = s_{M-\mu+1}, \qquad (4)$$
$$A_{\mu,\nu} = s_{M-\mu+1-\nu}. \qquad (5)$$

of linear equations.
General choice of the parameters n and m can make (3) over– or underdetermined, thus solving of

$$\mathbf{A}^{\mathrm{T}}\mathbf{A}\,\vec{a} = \mathbf{A}^{\mathrm{T}}\,\vec{b} \qquad (6)$$

for $\vec{a}$ is the appropriate method for parameter estimation.
Since $\mathbf{A}^{\mathrm{T}}\mathbf{A}$ is symmetric and relatively small (e.g. 200x200 Elements), many direct or iterative numeric solvers are available to treat (6) with.
A Signal Theory approach [4] using the signals covariance leads to the same result (6), but the approach presented here is simpler and can be easily optimized if $m = 2 * n$ by directly solving (3).

III. Frequency domain DFT/AR evaluation

The standard AR(n) method solves the explicit difference equation (1) in the time domain, using $s_{M-\nu}$, $\nu \in [1, n]$ as initial values. This way, the signal can be extrapolated for $M < k <= M + P$. The completed signal s_k, $k \in [0, K + P]$ is then transformed to the frequency domain using the well–known Discrete Fourier Transform DFT($M + P$).
If the AR(n) system yields instable poles, which can be observed for relatively large orders n, this algorithm does not work, since

DFT does not converge with exponentially raising time domain signals.

The DAR(n) method presented here uses a DFT(M) for frequency domain transformation of the FDTD calculated part of the time series.

The Difference Equation

$$s'_{k'} \quad = \quad \sum_{\nu=1}^{n} a_\nu s'_{k'-\nu}, \quad k' \in [0, \infty] \tag{7}$$

with $s_{k'+M+1} = s'_{k'}$ is now solved using the z–Transform

$$S(z) \quad = \quad \sum_{k'=0}^{\infty} z^{-k'} s'_{k'}, \tag{8}$$

with the initial values s'_λ, $\lambda \in [-n, -1]$.
Straight forward calculations lead to

$$S(z) \quad = \quad \frac{\sum_{\nu=1}^{n} a_\nu \sum_{\lambda=-n}^{-1} s'_\lambda z^{-(\nu+\lambda)}}{1 - \sum_{\nu=1}^{n} a_\nu z^{-\nu}}. \tag{9}$$

The complete (DFT+AR) frequency domain result is

$$S(\omega) \quad = \quad \sum_{k=0}^{M} \mathrm{e}^{-\mathrm{j}\omega\Delta k} s_k + z^{-M-1} S(z)\big|_{z=\mathrm{j}\omega\Delta}, \tag{10}$$

where Δ is the (subsampled) time step.

IV. Numerical Investigation

Figure 1 sketches a disc–type dielectric resonator for KA–Band usage on an Al_2O_3 Substrate with a graded mesh FDTD grid. The mode of interest is the $TE_{1,0,\delta}$–Mode, thus for symmetry reasons only a quarter of the structure has to be simulated and magnetic walls are used for mirroring. Losses ($\tan\delta = 10^{-4}$) of the disc have been taken into account at 25 GHz as conductivity.

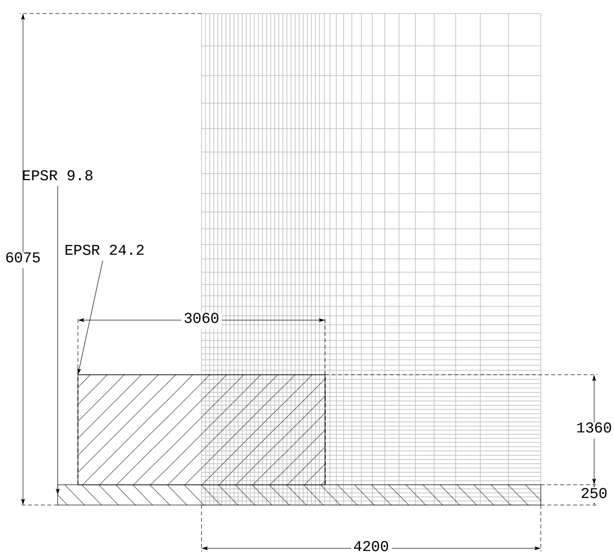

Fig. 1. Dielectric Resonator (ε_r = 24.2) on an Al_2O_3 substrate.

Figure 2 shows the first 500 samples (the subsample factor is 24, so 12000 FDTD time steps are shown) of the electric field component $E_\phi(t)$ in the middle of the resonator. Additive Excitation has been done with E_ϕ at another position, using a Gaussian pulse of 30 GHz bandwidth (-20 dB).

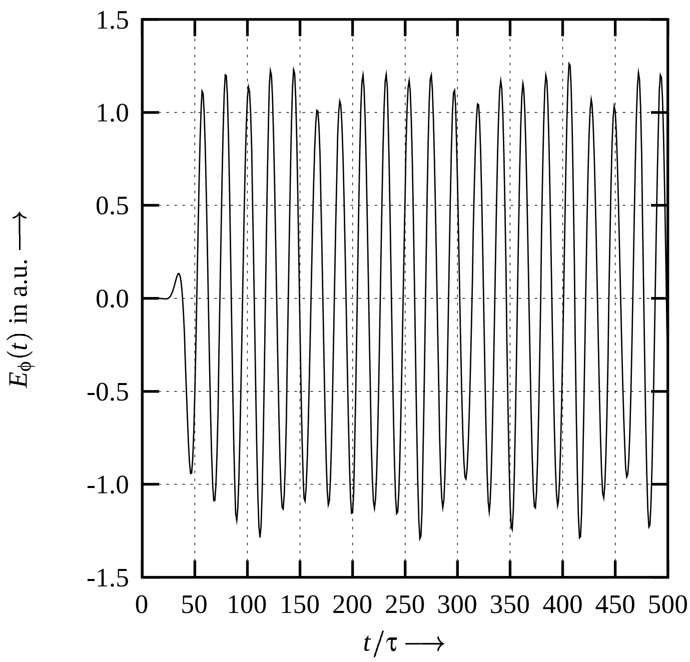

Fig. 2. Time domain test signal.

Figure 3 gives an impression of the transformed signal obtained with different algorithms. Here e.g. DAR(70,1/3) means n=70 and the first third of the samples has *not* been used to determine the system parameters (the transient part of the signal), so $m = M \cdot (1 - 1/3)$.

It can be seen that there are two resonances in the frequency range sketched and that DAR performs very much better as DFT.

To quantify the performance and do a comparison with the results of the standard AR method, the Resonance frequency (fig. 4 and fig. 5) calculated from the frequency domain data. The number of additional samples P for the AR method was 10000.

It can be stated, that

- The DAR method produces useable results for all orders ($n > 2$,$m > n$).
- The DAR Resonance frequency are *very* insensitive to the system order, if
  - the order n is chosen big enough
  - enough samples $m > 1.7 \cdot n$ are used for parameter estimation

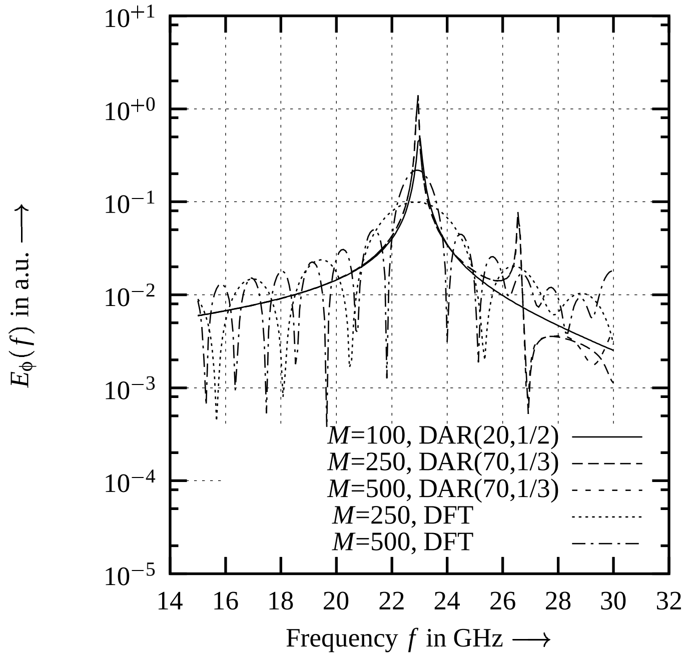

Fig. 3. Frequency domain test signal, calculated from 250 rsp. 500 Samples with the DFT rsp. DAR method.

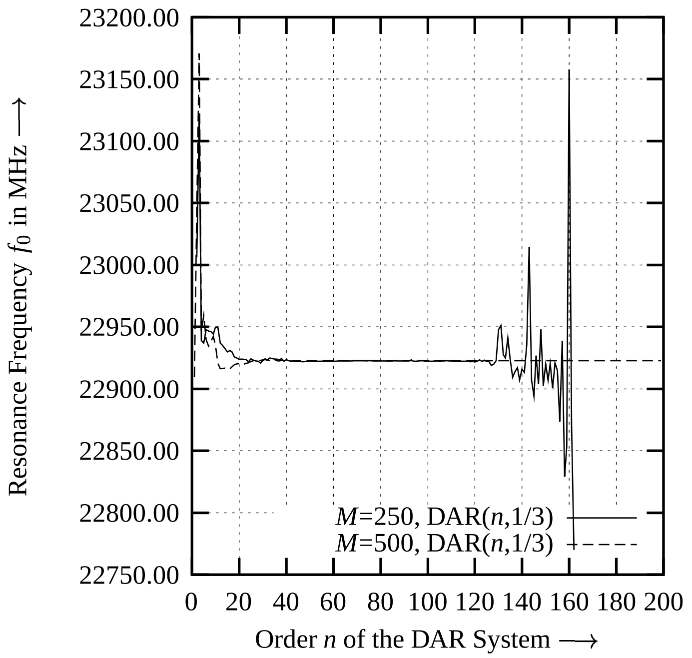

Fig. 4. Estimated Resonance frequency, calculated from 250 rsp. 500 Samples with the DAR(n,1/3) method. A 0.1 MHz grid was used for Resonance frequency search.

- The AR method had obvious instabilities (floating point overflows) for $n > 37$ at M = 250 samples and for $n \in [39, 93]$ and $n > 110$ at M = 500 samples.
- Hidden instabilities (exponential raise is too weak to cause floating point overflows at P=10000) lead to inexact results at some AR orders.
- Instable poles (same orders as AR instabilities) do *not* influence the DAR precision.

To show the DAR method's capabilities, more critical parameters have been derived from the frequency domain data. Figure 6 shows the estimated bandwidth, figure 7 the estimated signal strength.
It can be seen, that

- for precise bandwidth and signal strength M = 500 Samples are required.
- The DAR algorithm again works excellent, if the order n is big enough.

The AR algorithm's results have not been shown here, since

- Only $n \in [94, 110]$ for M = 500 Samples would be interesting here.
- Theoretically $P \approx 300,000$ extra samples would be necessary to achive the precision of DAR, making AR/DFT *very* slow.

The DAR algorithm is very fast, e. g. DAR(200,1/3) at M = 500 Samples took approx. 200 ms[1].

V. Advantages of DAR with regard to the SI method

Other signal models, like the System Identification (SI) method [2] have been sucessfully used for frequency domain signal estimation.
The main advantages of DAR with regard to the SI method are

- DAR's speed
- DAR does not use any iterative algorithm. So if the time domain signal changes slightly, the changes in the frequency domain signal are *not* caused by iterative algorithms deciding to stop after a slightly different number of iterations.

VI. Conclusions

A combined Autoregressive Signal model (AR)/DFT frequency domain method has been presented to overcome the AR stability problems. The algorithm is fast, its precision is excellent if the AR system's order is chosen big enough. Thus it is a very good choice for practical application.

[1] On a Pentium III(R) 450 MHz processor

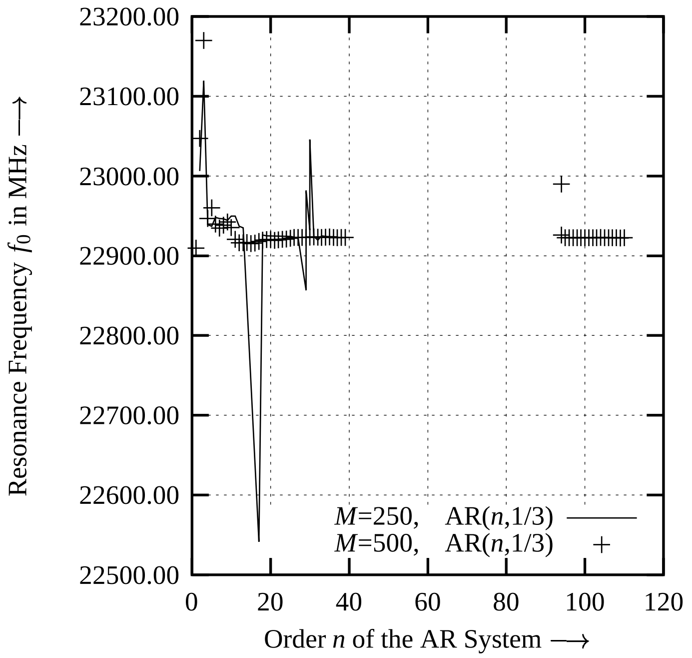

Fig. 5. Estimated Resonance frequency, calculated from 250 rsp. 500 Samples with the AR(n,1/3) method. A 0.1 MHz grid was used for Resonance frequency search.

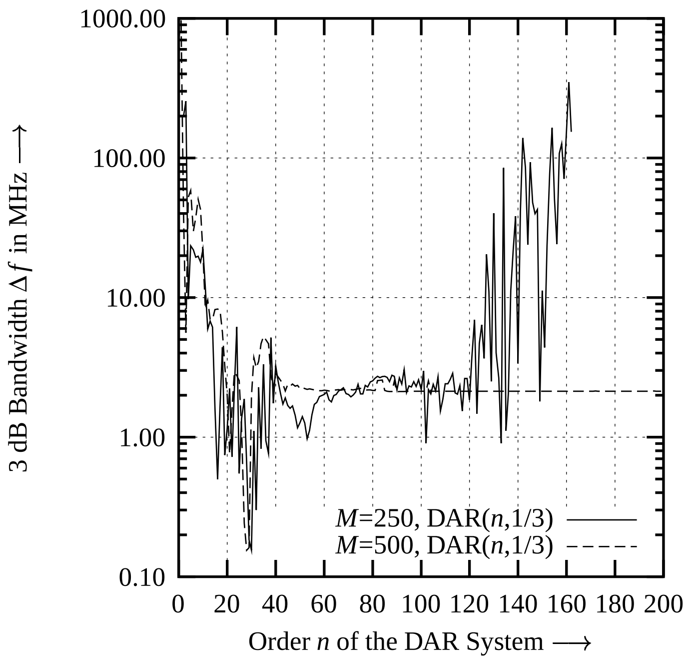

Fig. 6. Estimated 3dB Resonance Bandwidth, calculated from 250 rsp. 500 Samples with the DAR(n,1/3) method. A 0.1 MHz frequency step was used.

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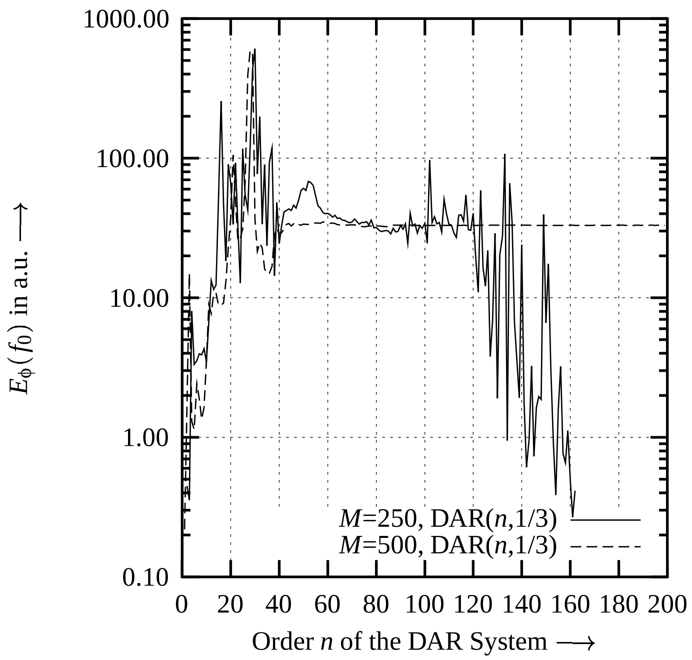

Fig. 7. Signal strength at the Resonance, calculated from 250 rsp. 500 Samples with the DAR(n,1/3) method.