ONE DIMENSIONAL CLUSTERED DIELECTRIC PARTICLE (CDP) PARALLEL-PLATE WAVEGUIDE (PPWG) STRUCTURE

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I. INTRODUCTION

Recently, metamaterials (MTMs) [1] have drawn considerable attention in the engineering and physics communities. Most of the MTMs reported to date are constituted of *metallic* inclusions periodically arranged in a host dielectric medium. Due to the presence of metal, which exhibits prohibitive losses at high frequencies, these MTMs are practically restricted to a frequency band below 100 GHz. In addition, metal-made structures, even at the low-end of the microwave range, are prone to short-circuiting and parasitic coupling with circuitry when used as a substrate in planar technologies. Therefore, purely dielectric MTM structures are strongly needed, to provide both optical MTMs and efficient microwave materials.

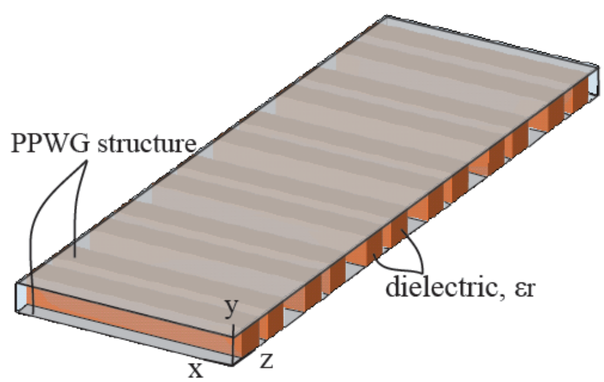
Conventional photonic crystals [2] can produce some MTM effects, such as negative refractive index (NRI) [3]. However, they generally exhibit very poor-quality refractive properties (e.g. very chaotic NRI focusing) because of their dominantly diffractive nature. For this reason, the concept of clustered dielectric particle (CDP) MTMs was recently introduced [4, 5]. A CDP MTM is constituted by the periodic repetition of a (molecule-like) cluster of dielectric (atom-like) particles with NRI properties, which, with appropriate cluster modes, can produce arbitrary positive and negative constitutive parameters over a restricted range of values.

In this paper, we examine a 1D parallel-plate waveguide (PPWG) CDP structure, propose an efficient transmission line method to compute its dispersion diagram, present a parametric study for its most meaningful parameters, and describe the left-handed (LH) backward-wave effects numerically observed in it in a high-frequency LH band. The motivation for this 1D CDP analysis is twofold. First, it should provide simpler (than in 2D/3D case) insight into the MTM phenomena. Second, it may lead to a diversity of components applications, such as those described in [1] for the case of metallic MTM structures.

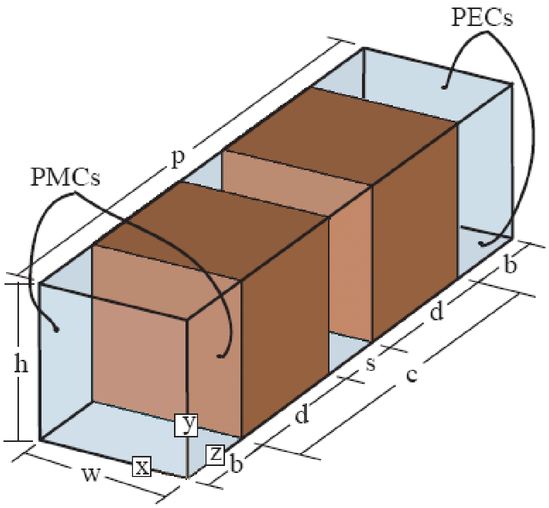
II. DESCRIPTION AND PRINCIPLE

The proposed CDP PPWG structure is shown in Fig. 1. It is the 1D counterpart of the 2D CDP MTMs shown in [4, 5], where NRI index phenomena were observed numerically. Interestingly, these refractive effects were obtained even in higher bands (3-6), where usually diffraction dominates propagation. This "artificial homogeneity" was shown to be a result of cluster modes with rotational symmetry. For easier computation, PMC walls were placed at both sides in the z direction, which emulates the behavior of a z-unbounded PPWG with perfect TEM transmission in the dominant mode. The unit cell is composed of a cluster or pair of two dielectric slabs, which means that we have a configuration similar to a photonic crystal with a biatomic cell.

The clusters will be shown to produce appropriate electric field ***E*** and magnetic field ***H*** loops so as to generate equivalent magnetic dipoles ***m*** and electric dipoles ***p***, respectively, which can lead to MTM effective constitutive parameters. In particular, these parameters should be negative if the dipole moments are opposite to the excitation (dia-magnetic/electric) and stronger than the relative parameters of the host medium.



(a)



(b)

Fig. 1: 1D CDP PPWG structure. The propagation direction is *z*. (a) Perspective view. (b) Unit cell (PECs on top and bottom sides perpendicularly to *y*, PMCs on left and right sides perpendicularly to *x*.).

III. TRANSMISSION LINE APPROACH

The dispersion diagram contains the most important information on the properties of a structure. Since the computation of such a diagram by full-wave analysis might be lengthy, we propose here an alternative simple and very fast transmission line approach. As shown in Fig. 2, each region of air or dielectric is modeled by a transmission line section with corresponding propagation constant and characteristic impedance.

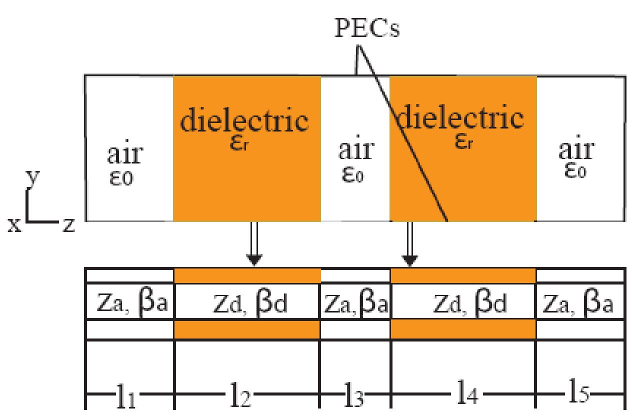


Fig. 2: Transmission line model of the unit cell of the 1D CDP PPWG structure

Due to TEM nature of the PPWG dominant mode, the propagation constant and characteristic impedance of each section k (*k = 1…5*) is simply given by

$$\beta_k = \sqrt{\varepsilon_{r\_k}\mu_{r\_k}}\,\frac{\omega}{c}, \qquad (1)$$

where *c* = speed of light in free space = 2.997925×10^8 m/s and

$$Z_{0k} = \sqrt{\frac{\mu_k}{\varepsilon_k}\frac{h}{w}}\,, \qquad (2)$$

where

$$\varepsilon_k = \varepsilon_0\varepsilon_{r\_k}\,, \qquad (3)$$

with

$$\varepsilon_{r\_1} = \varepsilon_{r\_3} = \varepsilon_{r\_5} = 1 \text{ (air)},$$
$$\varepsilon_1 = \varepsilon_3 = \varepsilon_5 = \varepsilon_0\,,$$
$$\varepsilon_{r\_2} = \varepsilon_{r\_4} = \varepsilon_r \text{ (dielectric)},$$
$$\varepsilon_2 = \varepsilon_4 = \varepsilon_0\varepsilon_r\,,$$
$$\mu_k = \mu_0\mu_{r\_k} = \mu_0, \text{ because } \mu_{r\_k} = 1, \forall k\,,$$
$$l_1 = l_5 = b,\ l_2 = l_4 = d \text{ and } l_3 = s\,.$$

The ABCD matrix for each section *k* reads

$$\begin{pmatrix} A_k & B_k \\ C_k & D_k \end{pmatrix} = \begin{pmatrix} \cos(\beta_k l_k) & jZ_{0k}\sin(\beta_k l_k) \\ jY_{0k}\sin(\beta_k l_k) & \cos(\beta_k l_k) \end{pmatrix}$$

while the ABCD matrix of the overall unit cell is obtained by multiplying the ABCD matrixes of the five cells

$$\begin{pmatrix} A & B \\ C & D \end{pmatrix}_5 = \prod_{k=1}^{5}\begin{pmatrix} A_k & B_k \\ C_k & D_k \end{pmatrix}, \quad k = 1,2,3,4,5\,. \qquad (4)$$

Applying Floquet theorem with periodicity *p*,

$$\begin{pmatrix} A & B \\ C & D \end{pmatrix}_5 \begin{pmatrix} V_{in} \\ I_{in} \end{pmatrix} = e^{+\gamma p}\begin{pmatrix} V_{in} \\ I_{in} \end{pmatrix} \overset{\text{(lossless)}}{=} e^{+j\beta p}\begin{pmatrix} V_{in} \\ I_{in} \end{pmatrix}, \qquad (5)$$

and setting the determinant of the resulting system

$$\begin{pmatrix} A - e^{+j\beta p} & B \\ C & D - e^{+j\beta p} \end{pmatrix}\begin{pmatrix} V_{in} \\ I_{in} \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix} \qquad (6)$$

to zero for a non-trivial solution, we obtain the following characteristic equation

$$AD - (A+D)e^{+j\beta p} + e^{+2j\beta p} - BC = 0\,, \qquad (7)$$

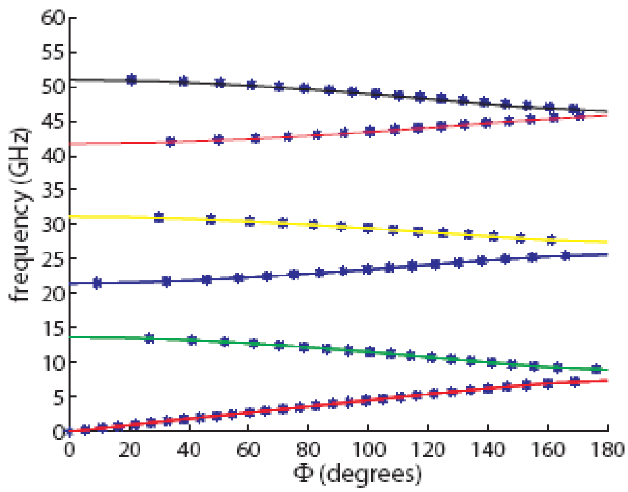
whose roots provide the dispersion relation β(ω). The ABCD matrix in (4) may be expanded analytically, but is more conveniently computed numerically.

IV. PARAMETRIC ANALYSIS

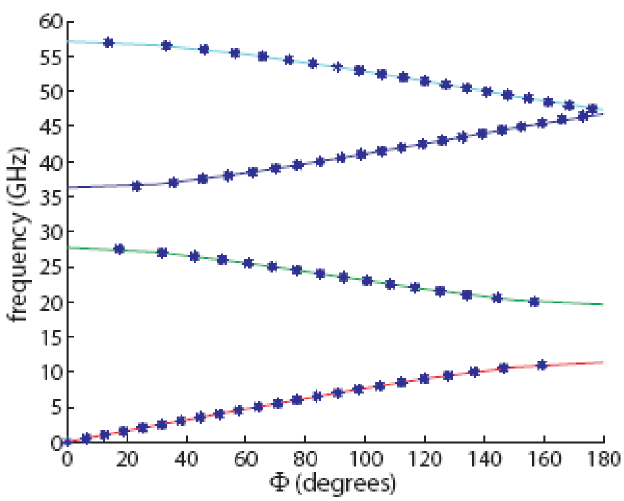
We perform now a parametric analysis of the dispersion diagrams obtained by the approach of Sec. III and compared with full-wave simulation results for the parameters indicated in Tab. 1. The corresponding results are presented in Fig. 3. Perfect agreement between the transmission line approach results and the full-wave results may be observed.

Table 1: Design parameters for the unit cell with $p = 7.5$ mm, $w = h = 2$ mm.

	Design #1	**Design #2**	**Design #3**
d (mm)	2	**0.4**	0.4
b (mm)	1.25	**3.25**	3.25
s (mm)	1	**0.2**	0.2
ε_r	10.2	10.2	**50**



(a)



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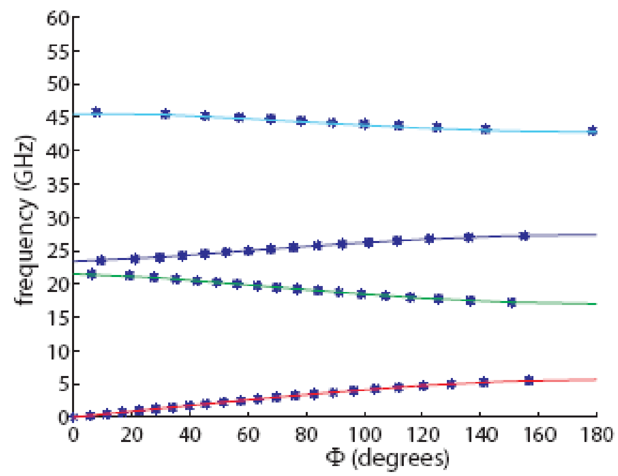

(c)

Fig, 3: Dispersion diagrams for the parameters indicated in Tab. 1. (*): transmission line approach (Sec. III), (-) full-wave (CST Microwave Studio, FDTD) transmission line. (a) Design #1 [first 6 modes]. (b) Design #2 [first 4 modes]. (c) Design #3 [first 4 modes].

Design #1 uses an inexpensive commercial substrate with parameters than can be achieved practically for an experimental prototype. The corresponding dispersion diagram (Fig. 3a) exhibits 3 positive- and 3 negative-gradient curves in the range from DC to 55 GHz. However, the size of the cluster is relatively large compared to period (c / p = 0.67). Consequently, the cluster cannot be rigorously assimilated to a molecule with simple dipole moments.

For this reason, a much smaller unit cell (c / p = 0.13) is considered in Design #2. Since the resulting volume of dielectric is smaller, the effective permittivity of the overall unit cell is decreased, which shifts the full band structure to higher frequencies (Fig.3b).

In order to restore the modes at about their initial location, Design #3 uses a higher dielectric constant (ε_r = 50) while maintaining the various dimensions unchanged (Fig. 3c).

V. BACKWARD-WAVE PROPAGATION

As mentioned in Sec. II, the idea to obtain MTM effects, in particular left-handedness or backward-wave propagation, is to generate artificial magnetic and electric dipole moments from electric and magnetic field loops, respectively. Such effects were observed in higher (than the first) bands in [4, 5] for 2D structures. In the case of the proposed PPWG structure, backward-wave propagation was observed in the 6th band for Design #1. The modal fields in this band are shown in Fig. 4. The electric field is seen to exhibit an anti-symmetric distribution with respect to the center of the unit cell; this leads to a displacement current loop generating an artificial magnetic dipole moment. Similarly the magnetic field exhibits two induction current loops, from a double anti-symmetric configuration, generating two electric dipole moments. The presence of longitudinal electric and magnetic field components may also be considered as creating series capacitance and shunt inductance effects, in agreement with the transmission line description of LH MTMs [1]. The negative-gradient curve is believed to be associated with the anti-symmetric field configurations, which lead to negative parameters. This is confirmed by the fact that the lower negative-gradient modes also exhibit anti-symmetric field distributions.

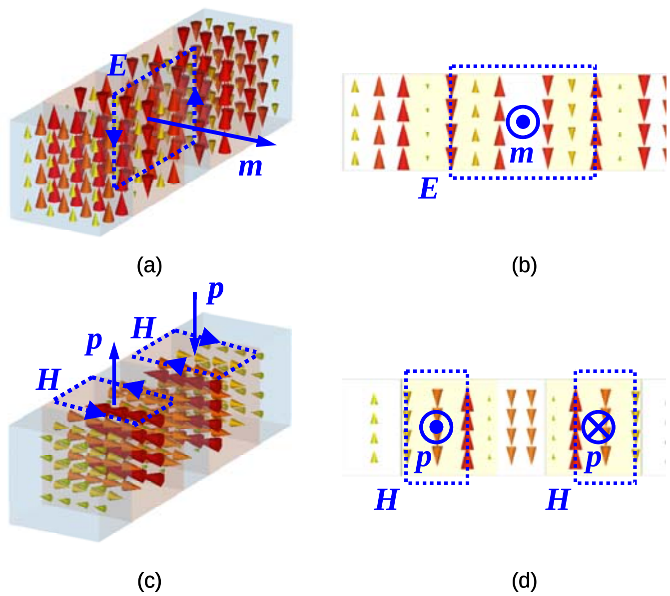

Fig. 4: Modal field distributions in Design #1 at the point Φ = 10° for the mode 6 (50.96 GHz) in the dispersion diagram of Fig. 3a.

Fig. 5 shows the backward-wave effect obtained in the 6th mode of Design #1, which can be nicely observed in a movie animation. Here the negative phase velocity is associated with the envelope of the waveform, which constitutes the *macroscopic propagating* variations of the wave, while spurious *microscopic standing-wave* variations are also visible. It is expected that these spurious variations could be reduced and suppressed by reducing the size of the unit cell, as done in Designs #2 and #3.

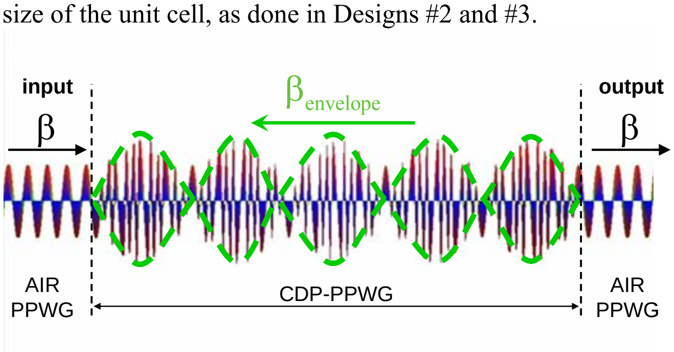

Fig. 5: Envelope backward-wave propagation observed for the (negative-gradient) mode of Fig. 4 in a 20-cell (i.e. 20 clusters or 40 dielectric elements) CDP PPWG structure. The simulation was performed with IMST Empire (FDTD).

VI. CONCLUSION

A 1D PPWG CDP structure was presented, along with an efficient transmission line method to compute its dispersion diagram. This structure was characterized parametrically in terms of its dispersion diagram. LH backward-wave effects were numerically observed in the 6th band of the structure.

Successful implementation of 1D CDP waveguides may lead to a wealth of optical components, following the concepts and ideas demonstrated with metallic MTM structures in the microwave regime [1].

VII. REFERENCES

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